MAA AMC
American Mathematics Competitions

MAA American Mathematics Competitions

35th Annual

AMC 8

Tuesday, November 12, 2019 through Monday, November 18, 2019

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.

2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.

3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.

4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.

5. Only blank scratch paper, blank graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the competition, your competition manager will ask you to record your name and other information on the answer sheet.

8. You will have 40 minutes to complete the competition once your competition manager tells you to begin.

9. When you finish the competition, sign your name in the space provided on the answer sheet.

1. Ike and Mike go into a sandwich shop with a total of $30.00 to spend. Sandwiches cost $4.50 each and soft drinks cost $1.00 each. Ike and Mike plan to buy as many sandwiches as they can and use any remaining money to buy soft drinks. Counting both soft drinks and sandwiches, how many items will they buy?

   (A) 6   **(B)** 7   **(C)** 8   **(D)** 9   **(E)** 10

2. Three identical rectangles are put together to form rectangle $ABCD$, as shown in the figure below. Given that the length of the shorter side of each of the smaller rectangles is 5 feet, what is the area in square feet of rectangle $ABCD$?
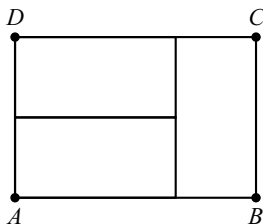

   (A) 45   **(B)** 75   **(C)** 100   **(D)** 125   **(E)** 150

3. Which of the following is the correct order of the fractions $\frac{15}{11}$, $\frac{19}{15}$, and $\frac{17}{13}$, from least to greatest?

   (A) $\frac{15}{11} < \frac{17}{13} < \frac{19}{15}$ **(B)** $\frac{15}{11} < \frac{19}{15} < \frac{17}{13}$ **(C)** $\frac{17}{13} < \frac{19}{15} < \frac{15}{11}$

   (D) $\frac{19}{15} < \frac{15}{11} < \frac{17}{13}$ **(E)** $\frac{19}{15} < \frac{17}{13} < \frac{15}{11}$

4. Quadrilateral $ABCD$ is a rhombus with perimeter 52 meters. The length of diagonal $\overline{AC}$ is 24 meters. What is the area in square meters of rhombus $ABCD$?
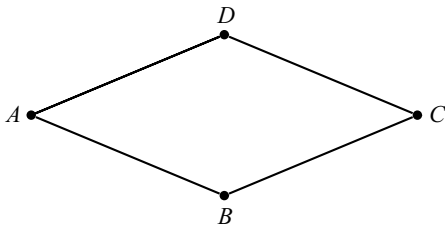
   (A) 60   **(B)** 90   **(C)** 105   **(D)** 120   **(E)** 144

5. A tortoise challenges a hare to a race. The hare eagerly agrees and quickly runs ahead, leaving the slow-moving tortoise behind. Confident that he will win, the hare stops to take a nap. Meanwhile, the tortoise walks at a slow steady pace for the entire race. The hare awakes and runs to the finish line, only to find the tortoise already there. Which of the following graphs matches the description of the race, showing the distance d traveled by the two animals over time t from start to finish?

(A)

(B)

(C)

(D)

(E)
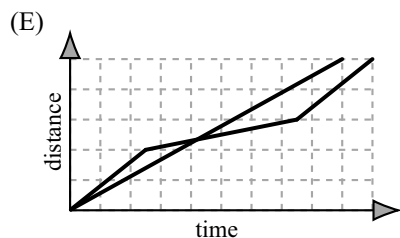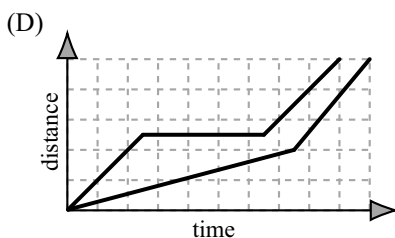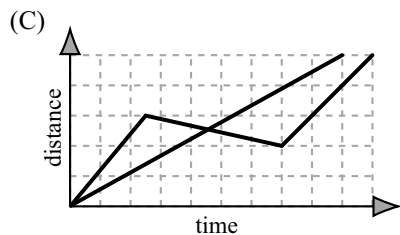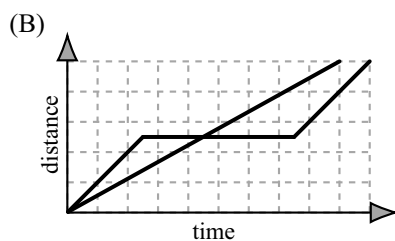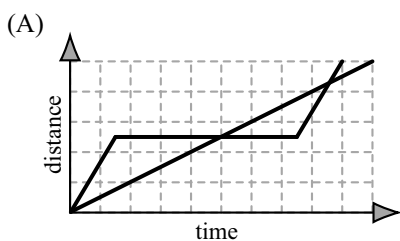
6. There are 81 grid points (uniformly spaced) in the square shown in the dia-
gram below, including the points on the edges. Point P is the center of the
square. Given that point Q is randomly chosen from among the other 80
points, what is the probability that the line PQ is a line of symmetry for the
square?
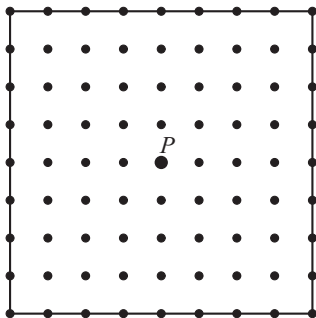
(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{9}{20}$ (E) $\dfrac{1}{2}$

7. Shauna takes five tests, each worth a maximum of 100 points. Her scores on
the first three tests are 76, 94, and 87. In order to average 81 for all five tests,
what is the lowest score she could earn on one of the other two tests?

(A) 48   (B) 52   (C) 66   (D) 70   (E) 74

8. Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then
Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives
25% of what is now left in the bag to her brother Jimmy. What percentage of
her original bag of marbles does Gilda have left for herself?

(A) 20   (B) $33\frac{1}{3}$ (C) 38   (D) 45   (E) 54

9. Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans
that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical
cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume
of one of Alex's cans to the volume of one of Felicia's cans?

(A) 1 : 4   (B) 1 : 2   (C) 1 : 1   (D) 2 : 1   (E) 4 : 1

10. The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?

Number of students at soccer practice
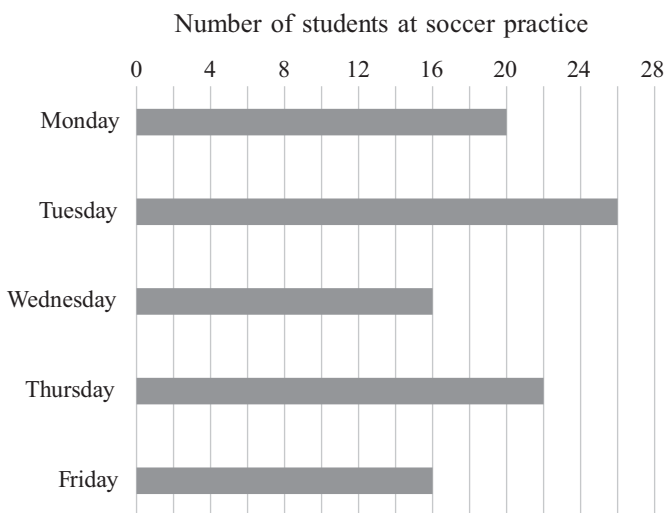
(A) The mean increases by 1 and the median does not change.
(B) The mean increases by 1 and the median increases by 1.
(C) The mean increases by 1 and the median increases by 5.
(D) The mean increases by 5 and the median increases by 1.
(E) The mean increases by 5 and the median increases by 5.

11. The eighth grade class at Lincoln Middle School has 93 students. Each student takes a math class or a foreign language class or both. There are 70 eighth graders taking a math class, and there are 54 eighth graders taking a foreign language class. How many eighth graders take *only* a math class and *not* a foreign language class?

(A) 16   (B) 23   (C) 31   (D) 39   (E) 70

12. The faces of a cube are painted in six different colors: red (R), white (W), green (G), brown (B), aqua (A), and purple (P). Three views of the cube are shown below. What is the color of the face opposite the aqua face?
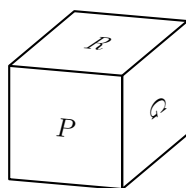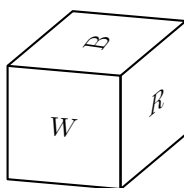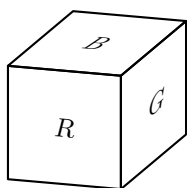
(A) red   (B) white   (C) green   (D) brown   (E) purple

13. A *palindrome* is a number that has the same value when read from left to right or from right to left. (For example 12321 is a palindrome.) Let N be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of N?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

14. Isabella has 6 coupons that can be redeemed for free ice cream cones at Pete's Sweet Treats. In order to make the coupons last, she decides that she will redeem one every 10 days until she has used them all. She knows that Pete's is closed on Sundays, but as she circles the 6 dates on her calendar, she realizes that no circled date falls on a Sunday. On what day of the week does Isabella redeem her first coupon?

(A) Monday   (B) Tuesday   (C) Wednesday   (D) Thursday

(E) Friday

15. On a beach 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap?

(A) $\dfrac{14}{85}$ (B) $\dfrac{7}{25}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{4}{7}$ (E) $\dfrac{7}{10}$

16. Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip?

    (A) 45   **(B)** 62   **(C)** 90   **(D)** 110   **(E)** 135

17. What is the value of the product

$$\left(\frac{1 \cdot 3}{2 \cdot 2}\right)\left(\frac{2 \cdot 4}{3 \cdot 3}\right)\left(\frac{3 \cdot 5}{4 \cdot 4}\right)\cdots\left(\frac{97 \cdot 99}{98 \cdot 98}\right)\left(\frac{98 \cdot 100}{99 \cdot 99}\right)?$$

    (A) $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{9800}{9801}$ **(D)** $\frac{100}{99}$ **(E)** 50

18. The faces on each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?

    (A) $\frac{4}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

19. In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams?

    (A) 22   **(B)** 23   **(C)** 24   **(D)** 26   **(E)** 30

20. How many different real numbers x satisfy the equation

$$\left(x^2 - 5\right)^2 = 16?$$

    (A) 0   **(B)** 1   **(C)** 2   **(D)** 4   **(E)** 8

21. What is the area of the triangle formed by the lines $y = 5$, $y = 1 + x$, and $y = 1 - x$?

    (A) 4   **(B)** 8   **(C)** 10   **(D)** 12   **(E)** 16

22. A store increased the original price of a shirt by a certain percent and then decreased the new price by the same percent. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased?

    (A) 16   **(B)** 20   **(C)** 28   **(D)** 36   **(E)** 40

23. After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?

    (A) 10   **(B)** 11   **(C)** 12   **(D)** 13   **(E)** 14

24. In triangle ABC, point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$?
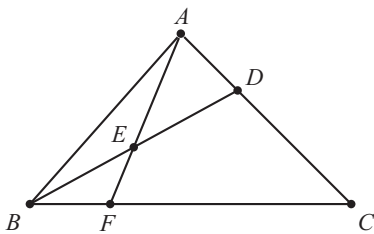
    (A) 24   **(B)** 30   **(C)** 32   **(D)** 36   **(E)** 40

25. Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

    (A) 105   **(B)** 114   **(C)** 190   **(D)** 210   **(E)** 380

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Scores and official competition solutions will be sent to your competition manager who can share that information with you.

For more information about the MAA American Mathematics Competitions program and our other competitions, please visit maa.org/amc.

Questions and comments about this competition should be sent to:

amcinfo@maa.org

or

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The problems and solutions for this AMC 8 were prepared by the MAA AMC 8 Editorial Board under the direction of:

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DO NOT OPEN UNTIL COMPETITION DAY.

The AMC 8 may be administered from Tuesday, November 12, 2019 through Monday, November 18, 2019.

Administration on an earlier date will disqualify your school's results.

- All the information needed to administer this competition is contained in the AMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE TUESDAY, NOVEMBER 12, 2019.

- Answer sheets must be returned to the MAA AMC office within 24 hours of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee timely arrival of these answer sheets. FedEx, UPS, or USPS overnight are strongly recommended.

- The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.